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Non-supersymmetric D1/D5, F/NS5 and closed string tachyon condensation

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Abstract

We construct the intersecting non-supersymmetric (non-susy) D1/D5 solution of type IIB string theory. While, as usual, the solution is charged under an electric two-form and an electric six-form gauge field, it also contains a non-susy chargeless (non-BPS) D0-brane. The S-dual of this solution is the non-susy F/NS5 solution. We show how these solutions nicely interpolate between the corresponding black (or non-extremal) solutions and the Kaluza-Klein (KK) "bubble of nothing" (BON) by continuously changing some parameters characterizing the solutions from one set of values to another. We show, by a time symmetric general bubble initial data analysis, that the final bubbles in these cases are static and stable and the interpolations can be physically interpreted as closed string tachyon condensation. As special cases, we recover the transition of two charge black F-string to BON, considered by Horowitz, and also the transition from AdS_3 black hole to global AdS_3.

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1 Introduction

In an interesting paper it has been argued by Horowitz [1] that under certain conditions black strings of type II string theories have dramatic new endpoints to Hawking evaporation in the form of KK BON [2][5]. He arrived at this conclusion by applying the closed string tachyonic instability to black strings. Closed string tachyons are known to develop when the fermions in the theory are taken to satisfy antiperiodic boundary conditions along one of the compact directions and the size of the circle becomes of the order of string scale [4, 5]. Adams et. al. [6] have argued that when these winding string tachyons are localized they can trigger a topology changing transformation as a consequence of the closed string tachyon condensation. The transition from the black string to the KK BON is an application of this process. A similar transition also occurs for black Dp-branes and was briefly mentioned in [1]. However, it was observed that only for $p = 3$ the final bubble could be static and stable. In ref.[7] we showed that this is not quite right and in fact, black Dp-branes can make transitions to stable static bubbles for all $p \leq 4$ via closed string tachyon condensation but in an indirect way as specified there for $p \neq 3$. We have also argued how this stringy process can be modelled by a series of classical supergravity configurations.

In [1] Horowitz started from the standard black fundamental string solution [8, 9] and compactified the string direction. Since here the metric is multiplied appropriately with a harmonic function, the size of the string wound along the compact direction varies monotonically from L (where L is the periodicity of the compact direction) to zero as we move along the radial direction from infinity to the singular point. So, at some point in between the size of the circle becomes of the order of string scale and if this occurs on the horizon, then the closed string tachyon condensation causes the circle to pinch off and the resulting state is a bubble which in this case cannot be static but should expand out. Static bubble appeared while considering a similar transition for a toroidally compactified black F-string solution containing both F and NS5 charge. However, in order to show that bubble is the end state of this transition, a time symmetric bubble initial data analysis has been performed and it was found that indeed under certain conditions ($Q/L^2 \ll 1$, where Q is the flux associated with the bubble) the bubble can be stable and so, the black F-string in those cases can make a transition to classically stable static bubbles. In this analysis the two charges of the black F-string were made equal, for simplicity, which in turn, decouples the dilaton in the solution.

In this paper we will construct the non-susy F/NS5 solution of the ten dimensional

[5] This has also been generalized to the $p = 0$ case, i.e., charged or uncharged black hole case, in [3].

type IIB string theory with two unequal charges and a non-trivial dilaton, generalizing the solution considered by Horowitz. In order to construct this solution we will start from the known type IIA solution representing intersections of charged non-susy[6] D0-brane and charged non-susy D4-branes given in [10]. We next obtain a delocalized (in one of the transverse directions of D4-brane) version of this solution which will introduce an isometry direction and then take T-duality along that direction. Note that since we are dealing with non-susy solution the procedure of delocalization and the resulting T-dual configurations are quite different from the usual BPS solutions we are familiar with. In this case (as opposed to BPS case) after T-duality, we obtain the intersecting charged non-susy D1/D5 solution and chargeless non-susy or non-BPS D0-branes. The S-dual of this solution is the intersecting non-susy F/NS5 solution. This solution can be interpreted as intersecting charged non-susy F/NS5 solution along with chargeless non-susy or non-BPS D0-branes. Both these solutions are characterized by six independent parameters. We will see that when these parameters are varied from one set of values to another keeping the physical conserved quantities such as the mass and the charges unchanged, these solutions nicely interpolate between black solutions and KK BON.

We emphasize that the existence of interpolating classical solutions does not necessarily imply that the transition from black brane to KK BON will actually occur (as happened for D5- and D6-branes). As we mentioned, in order to have a transition, we must ensure that the final bubble configuration is locally stable so that it does not evolve further perturbatively and this is done by the time symmetric general bubble initial data analysis. To show that the transition is caused by a perturbative process such as the closed string tachyon condensation, further conditions have to be satisfied. In particular, the curvature of the black brane near the horizon (where the closed string tachyon condensation occurs) must be much smaller than the string scale, otherwise the classical description breaks down and the black brane makes a transition to open string modes. Also, the horizon size and the bubble size, the charge of the black brane and the flux of the bubble and finally, the size of the compact circle at infinity of both the configurations must be equal. If all these requirements are satisfied, we can conclude that the black brane can make a transition to KK BON through closed string tachyon condensation. Conversely, if a black configuration makes a transition to KK BON via closed string tachyon condensation with all the restrictions being satisfied, the classical interpolating solution, if it exists, can be regarded as a model for the stringy process like the closed string tachyon condensation.

This is precisely what we will show in this paper. After constructing the interpolating solutions we will perform a time symmetric general bubble initial data analysis with

[6] Note that unlike the BPS branes the non-susy branes could be either charged or chargeless.

unequal charges and non-trivial dilaton (as opposed to the case considered by Horowitz) to show that the final bubble configuration can indeed be classically stable. This in turn implies that the interpolation means really a transition from black-brane to bubble. Then we carefully look at the various conditions mentioned earlier for the interpretation of this transition and whether it can be caused by a stringy process of closed string tachyon condensation and we find that indeed in certain cases this is true. The interpolating classical supergravity solutions can then be regarded as a model for this process. As a special case of non-susy F/NS5, the black-string to KK BON transition for the two charge black F-string in $D = 6$ considered by Horowitz can be understood and also from non-susy D1/D5, the AdS_3 black hole to AdS_3 soliton transition can be understood as a special case.

2 The interpolating solutions

In this section we will construct both the non-susy D1/D5 and F/NS5 solutions and show how by varying a subset of the parameters characterizing the solutions they nicely interpolate between the corresponding black or non-extremal solutions and the KK BON. For this purpose we start from the intersecting charged non-susy D$(p-4)$/Dp solution given in eqs.(1) – (6) of ref.[10] for $p = 4$. The corresponding charged non-susy D0/D4 solution is given as,

$$
\begin{aligned}
ds^2 &= F_2^{-\frac{3}{8}} F_1^{-\frac{7}{8}} (-dt^2) + F_2^{-\frac{3}{8}} F_1^{\frac{1}{8}} \sum_{i=1}^{4} (dx^i)^2 + \left(H\tilde{H}\right)^{\frac{2}{3}} F_2^{\frac{5}{8}} F_1^{\frac{1}{8}} \left(dr^2 + r^2 d\Omega_4^2\right), \\
e^{2\phi} &= F_2^{-\frac{1}{2}} F_1^{\frac{3}{2}} \left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \\
F_{[4]} &= b\mathrm{Vol}(\Omega_4), \qquad F_{[8]} = c\mathrm{Vol}(\Omega_4) \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4,
\end{aligned} \tag{1}
$$

where

$$
F_{1,2} = \cosh^2\theta_{1,2} \left(\frac{H}{\tilde{H}}\right)^{\alpha_{1,2}} - \sinh^2\theta_{1,2} \left(\frac{\tilde{H}}{H}\right)^{\beta_{1,2}} \tag{2}
$$

with $H = 1 + \omega^3/r^3$ and $\tilde{H} = 1 - \omega^3/r^3$, where $r = \sqrt{(x^5)^2 + \ldots + (x^9)^2}$. Note that the metric in (1) is given in the Einstein frame. (The metric in general would be given in the Einstein frame unless mentioned explicitly.) The solution is well-defined in the region $r > \omega$ and there is a singularity at $r = \omega$. Here $\alpha_{1,2}$, $\beta_{1,2}$, $\theta_{1,2}$, δ_1, ω are integration constants and b, c are the charge parameters, but not all the constants are independent. There are five relations among them given as follows,

$$
\alpha_1 - \beta_1 = -\frac{3}{2}\delta_1, \qquad \alpha_2 - \beta_2 = \frac{1}{2}\delta_1,
$$

$$b = 3(\alpha_2+\beta_2)\omega^3 \sinh 2\theta_2, \qquad c = 3(\alpha_1+\beta_1)\omega^3 \sinh 2\theta_1,$$
$$(\alpha_1+\beta_1)^2 + (\alpha_2+\beta_2)^2 + \frac{3}{2}\delta_1^2 = \frac{32}{3}. \tag{3}$$

By eliminating $\beta_{1,2}$ in the last relation of (3) we can rewrite it as,

$$\frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha_1(\alpha_1 + \frac{3}{2}\delta_1) + \frac{1}{2}\alpha_2(\alpha_2 - \frac{1}{2}\delta_1) = \frac{4}{3}. \tag{4}$$

So, the number of independent parameters characterizing the solution is five and we interpreted these parameters in [10] as the no. of D4-branes, no. of anti D4-branes, no. of D0-branes, no. of anti D0-branes and the tachyon parameter. Also note that in the solution (1) both the non-susy D4-branes and the non-susy D0-branes are magnetic. The corresponding electric solution will have the same form as (1) with the field-strengths $F_{[8]}$ and $F_{[4]}$ replaced by the electric gauge fields

$$\begin{aligned} A_{[1]} &= \frac{1}{2}\sinh 2\theta_1 \left(\frac{C_1}{F_1}\right) dt, \\ A_{[5]} &= \frac{1}{2}\sinh 2\theta_2 \left(\frac{C_2}{F_2}\right) dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 \end{aligned} \tag{5}$$

where $C_{1,2} = (H/\tilde{H})^{\alpha_{1,2}} - (\tilde{H}/H)^{\beta_{1,2}}$. Now if we want to have non-susy D1/D5 solution from here we first have to create an isometry direction along which we can take a T-duality transformation. For BPS branes this is usually done by placing the BPS branes in a periodic array along one of the transverse directions of the brane and then taking a continuum limit. This is possible due to the no-force condition of the BPS branes. For non-susy branes this procedure does not work and we have to obtain the delocalized solution directly by solving the equations of motion with a suitable ansatz. This was first done in [11] and then later in [12, 13]. So, from our experience we can write down the non-susy charged intersecting D0/D4 solution delocalized in one transverse direction as,

$$\begin{aligned} ds^2 &= F_2^{-\frac{3}{8}} F_1^{-\frac{7}{8}}(-dt^2) + F_2^{-\frac{3}{8}} F_1^{\frac{1}{8}} \sum_{i=1}^{4}(dx^i)^2 + F_2^{\frac{5}{8}} F_1^{\frac{1}{8}} \left(\frac{H}{\tilde{H}}\right)^{2\delta_2} (dx^5)^2 \\ &\qquad + \left(H\tilde{H}\right)\left(\frac{H}{\tilde{H}}\right)^{-\delta_2} F_2^{\frac{5}{8}} F_1^{\frac{1}{8}} \left(dr^2 + r^2 d\Omega_3^2\right), \\ e^{2\phi} &= F_2^{-\frac{1}{2}} F_1^{\frac{3}{2}} \left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \\ A_{[1]} &= \frac{1}{2}\sinh 2\theta_1 \left(\frac{C_1}{F_1}\right) dt, \\ A_{[5]} &= \frac{1}{2}\sinh 2\theta_2 \left(\frac{C_2}{F_2}\right) dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4, \end{aligned} \tag{6}$$

where $F_{1,2}$ remains the same as in (2), but H and $\tilde{H}$ have the forms $H = 1 + \omega^2/r^2$, $\tilde{H} = 1 - \omega^2/r^2$. Here $r = \sqrt{(x^6)^2 + \ldots + (x^9)^2}$. The parameter relations also remain the same[7] as in (3) except the last one (see the form in (4)) which takes the form,

$$\frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha_1(\alpha_1 + \frac{3}{2}\delta_1) + \frac{1}{2}\alpha_2(\alpha_2 - \frac{1}{2}\delta_1) = (1 - \delta_2^2)\frac{3}{2}. \tag{7}$$

The solution (6) represents intersecting non-susy D0/D4 solution delocalized in x^5 direction. So, x^5 is an isometry direction along which we will take a T-duality transformation to obtain the localized intersecting non-susy D1/D5 solution. Using the standard rules of T-duality transformation [14, 15, 16] on (6) we obtain,

$$\begin{aligned}
ds^2 &= \hat{F}_5^{-\frac{1}{4}}\hat{F}_1^{-\frac{3}{4}}\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{4}+\frac{\delta_2}{2}}(-dt^2) + \hat{F}_5^{-\frac{1}{4}}\hat{F}_1^{\frac{1}{4}}\left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{4}+\frac{\delta_2}{2}}\sum_{i=1}^{4}(dx^i)^2\\
&\quad + \hat{F}_5^{-\frac{1}{4}}\hat{F}_1^{-\frac{3}{4}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{3\delta_1}{4}-\frac{3\delta_2}{2}}(dx^5)^2 + \left(H\tilde{H}\right)\left(\frac{H}{\tilde{H}}\right)^{-\frac{\delta_1}{4}-\frac{\delta_2}{2}}\hat{F}_5^{\frac{3}{4}}\hat{F}_1^{\frac{1}{4}}\left(dr^2 + r^2 d\Omega_3^2\right),\\
e^{2\phi} &= \hat{F}_5^{-1}\hat{F}_1\left(\frac{H}{\tilde{H}}\right)^{2\delta_1 - 2\delta_2},\\
A_{[2]} &= \frac{1}{2}\sinh 2\theta_1\left(\frac{\hat{C}_1}{\hat{F}_1}\right) dt \wedge dx^5,\\
A_{[6]} &= \frac{1}{2}\sinh 2\theta_5\left(\frac{\hat{C}_5}{\hat{F}_5}\right) dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 \wedge dx^5,
\end{aligned} \tag{8}$$

where we have replaced the subscript '2' in the functions $\hat{F}$ by '5' to indicate that it is associated with the D5-brane. We have defined

$$\hat{F}_{1,5} = \cosh^2\theta_{1,5}\left(\frac{H}{\tilde{H}}\right)^{\hat{\alpha}_{1,5}} - \sinh^2\theta_{1,5}\left(\frac{\tilde{H}}{H}\right)^{\hat{\beta}_{1,5}} \tag{9}$$

with $\hat{C}_{1,5} = (H/\tilde{H})^{\hat{\alpha}_{1,5}} - (\tilde{H}/H)^{\hat{\beta}_{1,5}}$, and $H = 1 + \omega^2/r^2$, $\tilde{H} = 1 - \omega^2/r^2$. The functions $\hat{F}_{1,5}$ are related to the previous functions $F_{1,2}$ as follows,

$$\hat{F}_1 = F_1, \qquad \hat{F}_5 = \left(\frac{H}{\tilde{H}}\right)^{\frac{1}{2}\delta_1} F_2. \tag{10}$$

The parameter relations are now given as,

$$\begin{aligned}
&\hat{\alpha}_1 - \hat{\beta}_1 = -\frac{3}{2}\delta_1, \qquad \hat{\alpha}_5 - \hat{\beta}_5 = \frac{3}{2}\delta_1,\\
&(\hat{\alpha}_1 + \hat{\beta}_1)^2 + (\hat{\alpha}_5 + \hat{\beta}_5)^2 + \frac{3}{2}\delta_1^2 = (1 - \delta_2^2)12.
\end{aligned} \tag{11}$$

[7]For the magnetic solutions the parameter relations b, c will change as $b = 2(\alpha_2 + \beta_2)\omega^2 \sinh 2\theta_2$ and $c = 2(\alpha_1 + \beta_1)\omega^2 \sinh 2\theta_1$.

where the relations between the old parameters and the 'hatted' new parameters are given as,

$$\hat{\alpha}_5 = \alpha_2 + \frac{1}{2}\delta_1, \quad \hat{\alpha}_1 = \alpha_1, \quad \hat{\beta}_5 = \beta_2 - \frac{1}{2}\delta_1, \quad \hat{\beta}_1 = \beta_1. \tag{12}$$

By inspection it is clear from (8), that since the coefficient of $(-dt)^2$ is different from that of $\sum_{i=1}^4 (dx^i)^2$, the solution contains chargeless D0-brane. Also since $A_{[2]}$ and $A_{[6]}$ are non-zero the solution contains charged non-susy D-string lying along x^5 as well as charged non-susy D5-branes lying along $x^1, \ldots, x^5$ directions and so, (8) represents intersecting charged non-susy D1/D5 system with chargeless non-susy D0-brane. The solution (8) has six independent parameters, for example, one such independent set is ω, $\theta_{1,5}$, $(\hat{\alpha}_{1,5} + \hat{\beta}_{1,5})$, and δ_2.

To verify the correctness of the solution (8), we can check some special cases. For example, if we put $\delta_1 = -2\delta_2$, then (8) reduces to intersecting non-susy D1/D5 brane system obtained in [10]. Also redefining the paremeters as,

$$\hat{\alpha}_5 = \frac{3}{2}\bar{\delta}_1 - 2\delta_0 + \bar{\alpha}, \quad \hat{\alpha}_1 = -\bar{\delta}_1 - 2\delta_0, \quad \delta_1 = \bar{\delta}_1 - \frac{8}{3}\delta_0, \quad \delta_2 = \bar{\delta}_2 + \frac{4}{3}\delta_0 \tag{13}$$

and putting $\theta_1 = 0$ along with the redefinition $\hat{F}_5 = \bar{F}_5 \left(H/\tilde{H}\right)^{\frac{3}{2}\bar{\delta}_1 - 2\delta_0}$, the above solution (8) reduces to charged non-susy D5 brane intersecting with chargeless D1 and D0 branes considered in [7] with the new function $\bar{F}_5 = (H/\tilde{H})^{\bar{\alpha}} \cosh^2\theta_5 - (\tilde{H}/H)^{\bar{\beta}} \sinh^2\theta_5$.

Now in order to obtain the non-susy F/NS5 solution we will apply the S-duality transformation to (8). S-duality will not change the Einstein frame metric, but will change the dilaton to its inverse. The RR gauge field $A_{[2]}$ will change to NSNS gauge field and since the S-dual of D5-brane is NS5-brane which is magnetic we have to take the Hodge dual of the field-strength of $A_{[6]}$ and that will be an NSNS 3-form. So, the solution will be given as,

$$\begin{aligned}
ds^2 &= \hat{F}_5^{-\frac{1}{4}} \hat{F}_1^{-\frac{3}{4}} \left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{4}+\frac{\delta_2}{2}} (-dt^2) + \hat{F}_5^{-\frac{1}{4}} \hat{F}_1^{\frac{1}{4}} \left(\frac{H}{\tilde{H}}\right)^{\frac{\delta_1}{4}+\frac{\delta_2}{2}} \sum_{i=1}^4 (dx^i)^2 \\
&\quad + \hat{F}_5^{-\frac{1}{4}} \hat{F}_1^{-\frac{3}{4}} \left(\frac{H}{\tilde{H}}\right)^{-\frac{3\delta_1}{4}-\frac{3\delta_2}{2}} (dx^5)^2 + \left(H\tilde{H}\right) \left(\frac{H}{\tilde{H}}\right)^{-\frac{\delta_1}{4}-\frac{\delta_2}{2}} \hat{F}_5^{\frac{3}{4}} \hat{F}_1^{\frac{1}{4}} \left(dr^2 + r^2 d\Omega_3^2\right), \\
e^{2\tilde{\phi}} &= \hat{F}_5 \hat{F}_1^{-1} \left(\frac{H}{\tilde{H}}\right)^{-2\delta_1+2\delta_2}, \\
B_{[2]} &= \frac{1}{2}\sinh 2\theta_1 \left(\frac{\hat{C}_1}{\hat{F}_1}\right) dt \wedge dx^5, \\
H_{[3]} &= b\mathrm{Vol}(\Omega_3).
\end{aligned} \tag{14}$$

Let us now make a coordinate transformation from the radial coordinate r to ρ as

$$r = \rho\left(\frac{1+\sqrt{f}}{2}\right), \qquad \text{with}, \quad f = 1 - \frac{4\omega^2}{\rho^2} \equiv 1 - \frac{\rho_0^2}{\rho^2}. \tag{15}$$

Using (15) we find $H/\tilde{H} = f^{-1/2}$. Then in terms of this new Schwarzschild-like coordinate we can rewrite the solution (14) as follows,

$$\begin{aligned}
ds^2_{\rm str} &= e^{\tilde{\phi}/2} ds^2 = G_1^{-1} f^{\frac{\hat{\alpha}_1}{2}+\frac{\delta_1}{8}-\frac{\delta_2}{2}}(-dt^2) + f^{\frac{\delta_1}{8}-\frac{\delta_2}{2}}\sum_{i=1}^{4}(dx^i)^2 \\
&\quad + G_1^{-1} f^{\frac{\hat{\alpha}_1}{2}+\frac{5\delta_1}{8}+\frac{\delta_2}{2}}(dx^5)^2 + G_5 f^{-\frac{\hat{\alpha}_5}{2}+\frac{3\delta_1}{8}+\frac{1}{2}}\left(\frac{d\rho^2}{f}+\rho^2 d\Omega_3^2\right), \\
e^{2\tilde{\phi}} &= G_5 G_1^{-1} f^{-\frac{\hat{\alpha}_5}{2}+\frac{\hat{\alpha}_1}{2}+\delta_1-\delta_2}, \\
B_{[2]} &= \frac{1}{2}\sinh 2\theta_1 \left(\frac{1-f^{\frac{\hat{\alpha}_1+\hat{\beta}_1}{2}}}{G_1}\right) dt\wedge dx^5, \qquad H_{[3]} = b\text{Vol}(\Omega_3).
\end{aligned} \tag{16}$$

Note here that we have written the metric in the string frame. The functions $G_{1,5}$ are defined as,

$$G_{1,5} = \hat{F}_{1,5} f^{\frac{\hat{\alpha}_{1,5}}{2}} = \cosh^2\theta_{1,5} - f^{\frac{\hat{\alpha}_{1,5}+\hat{\beta}_{1,5}}{2}}\sinh^2\theta_{1,5} \tag{17}$$

where $\hat{F}_{1,5}$ are as defined in (9), with $H/\tilde{H} = f^{-1/2}$. The parameter relations remain the same as given in (11). The charge parameter is given as $b = (1/2)(\hat{\alpha}_5 + \hat{\beta}_5)\rho_0^2 \sinh 2\theta_5$. This solution (16) represents the non-susy F/NS5 solution and is characterized by six independent parameters, namely, $(\hat{\alpha}_1 + \hat{\beta}_1)$, $(\hat{\alpha}_5 + \hat{\beta}_5)$, ρ_0, θ_1, θ_5 and δ_2. We will get the two-charge non-susy F-string solution as a special case of this solution when compactified on T^4 and will be discussed later.

From the parameter relations (11) it is clear that if we put

$$\hat{\alpha}_1 + \hat{\beta}_1 = 2, \qquad \hat{\alpha}_5 + \hat{\beta}_5 = 2 \tag{18}$$

such that the functions

$$G_{1,5} \to \bar{G}_{1,5} = 1 + \rho_0^2 \sinh^2\theta_{1,5}/\rho^2 \tag{19}$$

take the form of the usual harmonic functions and also put

$$\delta_2 = -1/3 \tag{20}$$

(which implies $\delta_1 = -4/3$, $\hat{\alpha}_1 = 2$, $\hat{\beta}_1 = 0$, $\hat{\alpha}_5 = 0$ and $\hat{\beta}_5 = 2$), then the solution (16) reduces to

$$\begin{aligned}
ds^2_{\rm str} &= \bar{G}_1^{-1}\left(-f dt^2 + (dx^5)^2\right) + \sum_{i=1}^{4}(dx^i)^2 + \bar{G}_5\left(\frac{d\rho^2}{f}+\rho^2 d\Omega_3^2\right), \\
e^{2\tilde{\phi}} &= \bar{G}_5 \bar{G}_1^{-1}, \\
B_{[2]} &= \left(1-\bar{G}_1^{-1}\right)\coth\theta_1 dt\wedge dx^5, \qquad H_{[3]} = b\text{Vol}(\Omega_3).
\end{aligned} \tag{21}$$

This is precisely the black F/NS5 solution. On the other hand, if we put

$$\hat{\alpha}_1 + \hat{\beta}_1 = 2, \qquad \hat{\alpha}_5 + \hat{\beta}_5 = 2, \qquad \text{and} \qquad \delta_2 = 1/3 \tag{22}$$

(which implies $\delta_1 = 4/3$, $\hat{\alpha}_1 = 0$, $\hat{\beta}_1 = 2$, $\hat{\alpha}_5 = 2$ and $\hat{\beta}_5 = 0$), then the solution (16) reduces to

$$\begin{aligned}
ds^2_{\text{str}} &= \bar{G}_1^{-1}\left(-dt^2 + f(dx^5)^2\right) + \sum_{i=1}^{4}(dx^i)^2 + \bar{G}_5\left(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2\right),\\
e^{2\tilde{\phi}} &= \bar{G}_5\bar{G}_1^{-1},\\
B_{[2]} &= \left(1 - \bar{G}_1^{-1}\right)\coth\theta_1 dt\wedge dx^5, \qquad H_{[3]} = b\text{Vol}(\Omega_3).
\end{aligned} \tag{23}$$

This is the F/NS5 KK BON solution. Here in order to avoid conical singularity at $\rho = \rho_0$, the coordinate x^5 must be periodic with period

$$L = 2\pi\rho_0\cosh\theta_1\cosh\theta_5. \tag{24}$$

It is therefore clear that (16) is the solution which interpolates between the black or non-extremal F/NS5 solution (21) and the KK BON solution (23), by continuously varying the parameters $\hat{\alpha}_{1,5}$, $\hat{\beta}_{1,5}$) and δ_2 characterizing the solution and there is no need to take the double Wick rotation.

Similar interpolation from the black D1/D5 configuration to KK BON can be shown from the general non-susy intersecting D1/D5 system with chargeless D0 brane solution given in (8). In order to show this we will first go to the Schrodinger-like coordinate using (15). In this new coordinate the solution (8) in the string frame takes the form,

$$\begin{aligned}
ds^2_{\text{str}} &= e^{\phi/2}ds^2 = G_1^{-\frac{1}{2}}G_5^{-\frac{1}{2}}f^{\frac{\hat{\alpha}_1}{4}+\frac{\hat{\alpha}_5}{4}-\frac{3\delta_1}{8}}(-dt^2) + G_1^{\frac{1}{2}}G_5^{-\frac{1}{2}}f^{\frac{\hat{\alpha}_5}{4}-\frac{\hat{\alpha}_1}{4}-\frac{3\delta_1}{8}}\sum_{i=1}^{4}(dx^i)^2\\
&\quad + G_1^{-\frac{1}{2}}G_5^{-\frac{1}{2}}f^{\frac{\hat{\alpha}_1}{4}+\frac{\hat{\alpha}_5}{4}+\frac{\delta_1}{8}+\delta_2}(dx^5)^2 + G_5^{\frac{1}{2}}G_1^{\frac{1}{2}}f^{-\frac{\hat{\alpha}_1}{4}-\frac{\hat{\alpha}_5}{4}-\frac{\delta_1}{8}+\frac{\delta_2}{2}+\frac{1}{2}}\left(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2\right),\\
e^{2\phi} &= G_5^{-1}G_1 f^{\frac{\hat{\alpha}_5}{2}-\frac{\hat{\alpha}_1}{2}-\delta_1+\delta_2},\\
A_{[2]} &= \frac{1}{2}\sinh 2\theta_1\left(\frac{1 - f^{\frac{\hat{\alpha}_1+\hat{\beta}_1}{2}}}{G_1}\right)dt\wedge dx^5,\\
A_{[6]} &= \frac{1}{2}\sinh 2\theta_1\left(\frac{1 - f^{\frac{\hat{\alpha}_5+\hat{\beta}_5}{2}}}{G_5}\right)dt\wedge dx^1\wedge dx^2\wedge dx^3\wedge dx^4\wedge dx^5.
\end{aligned} \tag{25}$$

The parameter relations are as given before in (11). The various functions appeared in the solution are as defined earlier. Now again we find that if we choose

$$\hat{\alpha}_1 + \hat{\beta}_1 = 2, \quad \hat{\alpha}_5 + \hat{\beta}_5 = 2, \quad \text{and} \quad \delta_2 = -\frac{1}{3} \tag{26}$$

implying (from (11)) $\delta_1 = -4/3$, $\hat{\alpha}_1 = 2$, $\hat{\beta}_1 = 0$ and $\hat{\alpha}_5 = 0$, $\hat{\beta}_5 = 2$, then the solution (8) reduces to,

$$
\begin{aligned}
ds^2_{\rm str} &= \bar{G}_1^{-\frac{1}{2}}\bar{G}_5^{-\frac{1}{2}}\left(-f dt^2 + (dx^5)^2\right) + \bar{G}_1^{\frac{1}{2}}\bar{G}_5^{-\frac{1}{2}}\sum_{i=1}^{4}(dx^i)^2 + \bar{G}_1^{\frac{1}{2}}\bar{G}_5^{\frac{1}{2}}\left(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2\right),\\
e^{2\phi} &= \bar{G}_5^{-1}\bar{G}_1,\\
A_{[2]} &= \left(1 - \bar{G}_1^{-1}\right)\coth\theta_1 dt \wedge dx^5,\\
A_{[6]} &= \left(1 - \bar{G}_5^{-1}\right)\coth\theta_5 dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 \wedge dx^5.
\end{aligned} \tag{27}
$$

Note here that for the above choice of parameters (26), $G_{1,5} \to \bar{G}_{1,5} = 1 + \rho_0^2 \sinh^2\theta_{1,5}/\rho^2$. This is precisely the black D1/D5 solution. On the other hand if we choose

$$
\hat{\alpha}_1 + \hat{\beta}_1 = 2, \quad \hat{\alpha}_5 + \hat{\beta}_5 = 2, \quad \text{and} \quad \delta_2 = \frac{1}{3} \tag{28}
$$

implying (from (11)) $\delta_1 = 4/3$, $\hat{\alpha}_1 = 0$, $\hat{\beta}_1 = 2$ and $\hat{\alpha}_5 = 2$, $\hat{\beta}_5 = 0$, then the metric in (8) reduces to,

$$
ds^2_{\rm str} = \bar{G}_1^{-\frac{1}{2}}\bar{G}_5^{-\frac{1}{2}}\left(-dt^2 + f(dx^5)^2\right) + \bar{G}_1^{\frac{1}{2}}\bar{G}_5^{-\frac{1}{2}}\sum_{i=1}^{4}(dx^i)^2 + \bar{G}_1^{\frac{1}{2}}\bar{G}_5^{\frac{1}{2}}\left(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2\right). \tag{29}
$$

The other fields remain the same as in (27). To avoid the conical singularity at $\rho = \rho_0$ the periodicity of x^5 coordinate remains the same as in (24). This is the corresponding KK BON. Again we see that by continuously varying the parameters $\hat{\alpha}_{1,5}$, $\hat{\beta}_{1,5}$, and δ_2 the solution (8) smoothly changes from the black D1/D5 solution to the KK BON.

3 Initial data analysis

In the previous section we obtained both non-susy F/NS5 (eq.(16)) and D1/D5 (eq.(25)) solutions characterized by six independent parameters. Further we have seen that when three of the parameters are varied continuously the solutions nicely interpolate between the corresponding non-extremal or black solutions and KK BON. Now in order to interpret this interpolation as a physical transition from the black solution to KK BON, we must ensure that the final bubble is perturbatively stable and static such that it does not evolve further. For this purpose we will perform a time symmetric general bubble initial data analysis. The time symmetric F/NS5 bubble metric (in Einstein frame) and the non-trivial dilaton have the forms,

$$
\begin{aligned}
ds^2 &= \bar{G}_1^{-\frac{3}{4}}\bar{G}_5^{-\frac{1}{4}} f(\rho)(dx^5)^2 + \bar{G}_1^{\frac{1}{4}}\bar{G}_5^{-\frac{1}{4}}\sum_{i=1}^{4}(dx^i)^2 + \bar{G}_1^{\frac{1}{4}}\bar{G}_5^{\frac{3}{4}}\left(\frac{d\rho^2}{f(\rho)h(\rho)} + \rho^2 d\Omega_3^2\right),\\
e^{2\tilde{\phi}} &= \bar{G}_5\bar{G}_1^{-1}
\end{aligned} \tag{30}
$$

where $\bar{G}_{1,5}$ and $f(\rho)$ were defined earlier and $h(\rho)$ is an unknown function to be determined from the constraint equation of the time symmetric initial data. The constraint obtained from the Einstein equation gives the solution of $h(\rho)$ in the form,

$$\begin{aligned} & h(\rho) - 1 \\ & = \frac{\lambda\left(\rho^2 + \rho_0^2 \sinh^2\theta_1\right)\left(\rho^2 + \rho_0^2 \sinh^2\theta_5\right)}{\rho^2\left[3\rho^4 + 2\rho_0^2\rho^2\left(\sinh^2\theta_1 + \sinh^2\theta_5 - 1\right) + \rho_0^4\left(\sinh^2\theta_1 \sinh^2\theta_5 - \sinh^2\theta_1 - \sinh^2\theta_5\right)\right]} \end{aligned} \tag{31}$$

where λ is an integration constant. This therefore gives a four parameter (λ, ρ_0, θ_1 and θ_5) family of time symmetric, asymptotically flat initial data. Note that the angles θ_1 and θ_5 are related to the charges associated with the F-strings and the NS5-branes as follows,

$$Q_{1,5} = \rho_0^2 \sinh 2\theta_{1,5}. \tag{32}$$

To avoid the conical singularity at $\rho = \rho_0$, the length of the x^5-circle at infinity must be

$$L = 2\pi\rho_0 \cosh\theta_1 \cosh\theta_5 \left(1 + \frac{\lambda}{\rho_0^2}\right)^{-\frac{1}{2}}. \tag{33}$$

The ADM mass of the bubble can be obtained from the metric in (30) as,

$$\begin{aligned} M \;=\; \frac{\Omega_3\rho_0^2}{2\kappa^2}\Bigg[&\sqrt{1 + \left(\frac{Q_1}{\rho_0^2}\right)^2} + \sqrt{1 + \left(\frac{Q_5}{\rho_0^2}\right)^2} \\ & - \frac{1}{4}\left(\frac{2\pi\rho_0}{L}\right)^2\left(1 + \sqrt{1 + \left(\frac{Q_1}{\rho_0^2}\right)^2}\right)\left(1 + \sqrt{1 + \left(\frac{Q_5}{\rho_0^2}\right)^2}\right)\Bigg] \end{aligned} \tag{34}$$

where $\Omega_3 = 2\pi^2$ is the volume of a unit 3-sphere and $2\kappa^2 = 16\pi G$, with G, the Newton's constant. Here (33) has also been used in eliminating the parameter λ. Note that for given $Q_{1,5}$ and L, the mass (34) takes the value $(\pi^2/\kappa^2)(Q_1 + Q_5 - \pi^2 Q_1 Q_5/L^2)$ for $\rho_0 \to 0$ and for $\rho_0 \to \infty$, $M \to -\infty$. So, there is no lower bound on mass and the positive energy theorem fails as was also noticed in [1]. The mass (34) has extremum when

$$\frac{dM}{d\rho_0} = 0 = \frac{\Omega_3\rho_0}{\kappa^2}\left(\frac{1}{\cosh 2\theta_1} + \frac{1}{\cosh 2\theta_5}\right)\left(1 - \frac{4\pi^2\rho_0^2}{L^2}\cosh^2\theta_1 \cosh^2\theta_5\right) \tag{35}$$

and this gives,

$$L = 2\pi\rho_0 \cosh\theta_1 \cosh\theta_5. \tag{36}$$

Comparing this with (33), we find that the extremum occurs at $\lambda = 0$ or $h(\rho) = 1$. The resulting metric in (30) is now the spatial part of the static bubble one obtains from the double Wick rotation of the black F/NS5 solution (21) in Einstein frame.

Now we will try to see whether the extremum is a local maximum or a local minimum by evaluating the double derivative $d^2M/d\rho_0^2$ and determining its sign. We find,

$$\left.\frac{2\kappa^2}{\Omega_3}\frac{d^2M}{d\rho_0^2}\right|_{\lambda=0}$$
$$= \frac{16\pi^2\rho_0^2}{L^2}\cosh 2\theta_1 \cosh 2\theta_5 \left(\frac{1}{\cosh 2\theta_1} + \frac{1}{\cosh 2\theta_5}\right)\left(1 - \frac{1}{\cosh 2\theta_1} - \frac{1}{\cosh 2\theta_5}\right). \quad (37)$$

However, it is clear that it is not easy to determine the position and the nature of the extremum from (36) and (37) along with (32) for given Q_1, Q_5 and L. So, we will look at the M-ρ_0 relation (34) more closely and try to find them in an indirect way.

For this purpose let us define two functions, based on (32) and (36), as follows,

$$\begin{aligned}
\frac{Q_5}{L^2} &= \frac{\sinh\theta_5}{2\pi^2\cosh\theta_5\cosh^2\theta_1} = \frac{\sinh\theta_5}{\pi^2(1+\sqrt{1+k^2\sinh^2 2\theta_5})\cosh\theta_5} \equiv C_5(\theta_5),\\
\frac{Q_1}{L^2} &= \frac{\sinh\theta_1}{2\pi^2\cosh\theta_1\cosh^2\theta_5} = \frac{\sinh\theta_1}{\pi^2(1+\sqrt{1+k^{-2}\sinh^2 2\theta_1})\cosh\theta_1} \equiv C_1(\theta_1) \qquad (38)
\end{aligned}$$

where $k = Q_1/Q_5 = \sinh 2\theta_1/\sinh 2\theta_5 \neq 0$. Note that for given Q_5, Q_1 and L (therefore given k), the solution of either $C_5(\theta_5)$ or $C_1(\theta_1)$ equation (they are correlated by k) above will give us the position of ρ_0 where the extremum occurs. This is because once we obtain θ_5 and θ_1, ρ_0 where extremum occurs can be determined from (32) or (36). Now to solve (38) we first note that $C_5(\theta_5)$ approaches zero for both $\theta_5 \to 0$ and $\theta_5 \to \infty$, while in between it is non-zero and positive. So, we expect at least one maximum for $C_5(\theta_5)$ for some $0 < \theta_5 < \infty$. The same is true for $C_1(\theta_1)$. In the following we will argue that there exists only one such maximum for either $C_5(\theta_5)$ or $C_1(\theta_1)$ (the two are correlated by $C_1(\theta_1) = kC_5(\theta_5)$). For concreteness we will focus on $C_5(\theta_5)$ only and give results for $C_1(\theta_1)$. Differentiating $C_5(\theta_5)$ in (38) with respect to θ_5 and putting it to zero we get,

$$\frac{dC_5(\theta_5)}{d\theta_5} = 0 \Rightarrow 16k^2x^4 + 16k^2x^3 - 4x - 1 = 0 \qquad (39)$$

where $x = \sinh^2\theta_5 \geq 0$. Let us define $G(x) = 16k^2x^4 + 16k^2x^3 - 4x - 1$. Then by looking at its behavior we can infer that there is a unique solution of the equation $G(x) = 0$. First note that $G(x = 0) = -1$ and $G(x \to \infty) \to \infty$. Further, $d^2G/dx^2 = 96k^2x(2x + 1) > 0$ for all $x > 0$ which means that $G(x)$ has a unique minimum in $0 < x < \infty$. Now since $G(x = 0) = -1 < 0$, $G(x)$ will cut the x-axis only once. So, it is clear that there

exists only one θ_5 (let us call this $\theta_{5\max}$) for which $G(x) = 0$. This $\theta_{5\max}$ must give the maximum of $C_5(\theta_5)$, denoted as $C_{5\max}$. So, given Q_5 and L if $Q_5/L^2 > C_{5\max}$, then there exists no static bubble, if $Q_5/L^2 = C_{5\max}$, the ADM mass has a turning point where the first and the second derivatives vanish[8] , while if $Q_5/L^2 < C_{5\max}$, there exist two static bubbles. In the last case, the solution with large θ_5, denoted as $\theta_{5\ell}$, will give small ρ_0, while the solution with small θ_5, denoted as θ_{5s}, will give large ρ_0 (see eq.(32)). So, we have $\theta_{5s} < \theta_{5\max} < \theta_{5\ell}$. Note that since ADM mass (eq.(34)) $M \to -\infty$ as $\rho \to \infty$, we expect large ρ_0 (θ_{5s}) corresponds to the maximum of mass (or unstable static bubble) while small ρ_0 ($\theta_{5\ell}$) corresponds to the minimum of mass (or stable static bubble).

Values of $C_{\max}$ and $\theta_{\max}$

Now let us make some estimate of the values of $C_{5,1\,\max}$ and $\theta_{5,1\,\max}$ discussed above. This will certainly depend on the value of the parameter $k = Q_1/Q_5 = \sinh 2\theta_1/\sinh 2\theta_5$. So, we will consider three different cases: (i) $k \sim 1$, (ii) $k \ll 1$ and (iii) $k \gg 1$.

Case (i): When $k \sim 1$, we can make an order of magnitude estimate of $C_{\max}$ and $\theta_{\max}$ by taking $k = 1$ for simplicity. In this case eq.(39) can be factorized as, $(2x-1)(2x+1)^3 = 0$, which can be solved to give $x = 1/2$ (note that $x \geq 0$). This in turn gives, $\sinh\theta_{5\max} = 1/\sqrt{2}$ and so, we get,

$$
\begin{aligned}
e^{\theta_{5\max}} &= \frac{1+\sqrt{3}}{\sqrt{2}} \approx 1.94, \\
C_{5\max} &= \frac{1}{3\sqrt{3}\pi^2} \approx 0.02.
\end{aligned} \tag{40}
$$

As we have argued, in order to have a stable static bubble we need to have $Q_5/L^2 < C_{5\max} \sim 0.02$ which is a small but fixed number. Also as $k \sim 1$, so, both $\theta_{1\max}$ and $C_{1\max}$ of the strings should be of the same order of magnitude as the corresponding quantities of NS5-branes given in (40).

Case (ii): When $k \ll 1$, eq.(39) simplifies to $16k^2(x^4 + x^3) = 1 + 4x$. It can be solved next to leading order as $x = (2k)^{-2/3}(1 - (2k)^{2/3}/4)$, which in turn gives, $\sinh\theta_{5\max} = (2k)^{-1/3}(1 - (2k)^{2/3}/8)$. We, therefore, have

$$
\begin{aligned}
e^{\theta_{5\max}} &= \frac{2}{(2k)^{\frac{1}{3}}}\left(1 + \frac{(2k)^{\frac{2}{3}}}{8}\right), \\
C_{5\max} &= \frac{1}{2\pi^2}\left(1 - \frac{3}{4}(2k)^{\frac{2}{3}}\right) \approx 0.05.
\end{aligned} \tag{41}
$$

[8]One can check that $G(x) = 0$ corresponds precisely to the vanishing second derivative.

So, we now have a larger $C_{5\max}$ and a very large $\theta_{5\max}$. Therefore, in this case the condition $Q_5/L^2 \ll C_{5\max}$ (which is necessary for the existence of stable static bubble as well as the occurrence of closed string tachyon condensation as we will see) can be easily satisfied. From (38) we find that $\sinh 2\theta_{1\max} = k \sinh 2\theta_{5\max} = (2k)^{1/3}$, which is vanishingly small for $k \ll 1$. This therefore tells us that even though black NS5-branes do not make transition to the stable static bubble via closed string tachyon condensation, addition of a few strings makes this transition possible.

Case (iii): In this case eq.(39) gives us the solution for x to the leading order as $x = 1/(2k)^{2/3}$. This implies $\sinh \theta_{5\max} = 1/(4k)^{1/3}$. Therefore, we have,

$$\begin{aligned} e^{\theta_{5\max}} &= 1 + \frac{1}{(4k)^{\frac{1}{3}}}, \\ C_{5\max} &= \frac{1}{2\pi^2 k}. \end{aligned} \tag{42}$$

So, $C_{5\max}$ is vanishingly small for $k \gg 1$ and as a result, it might seem that there is no possibility of a transition from black brane to stable static bubble (also the occurrence of closed string tachyon condensation) in this case. However, looking at eq.(38) we find that $\sinh 2\theta_{1\max} = k \sinh 2\theta_{5\max} = (4k)^{2/3}/2 \gg 1$. So, we have very large $\theta_{1\max}$ and also to leading order $C_{1\max} = 1/(2\pi^2) \approx 0.05$. This is good for the transition to the stable static bubble and the occurrence of closed string tachyon condensation. This case is just opposite to the previous case in the sense that strings in the previous case plays the role of NS5-branes and the NS5-branes in the previous case plays the roles of strings in this case. So, here, even though the black strings do not make a transition to stable static bubble via closed string tachyon condensation, addition of a few NS5-branes makes the transition possible.

Transition to stable static bubble

After having some estimate on the parameters $C_{5,1\max}$ and $\theta_{5,1\max}$, we will try to see under what condition the black configuration will make a transition to stable static bubble (when the parameter $\theta_{5,1}$ takes values $\theta_{5,1\ell}$ corresponding to small bubble) and what is the estimate for the values of $\theta_{5,1\ell}$. This will be necessary for the interpretation of the interpolating solution as the process of closed string tachyon condensation and will be discussed in the next section.

We have seen that for $k \sim 1$, when $Q_5/L^2 \sim Q_1/L^2 < C_{5\max} \sim C_{1\max} \approx 0.02$, there exist two static bubbles, of which the smaller one (smaller ρ_0 which corresponds to bigger θ i.e. $\theta_{5,1\ell}$) corresponds to the stable static bubble. Further, the closed string tachyon

condensation requires (as we will discuss later) $Q_1/L^2 \sim Q_5/L^2 \ll 0.02$ and so, we get from (38),

$$e^{\theta_{1\ell}} \sim \sqrt{\frac{2}{Q_5}}\frac{L}{\pi}, \qquad e^{\theta_{5\ell}} \sim \sqrt{\frac{2}{Q_1}}\frac{L}{\pi}. \tag{43}$$

On the other hand, for $k \ll 1$, when $Q_5/L^2 < C_{5\max} \approx 0.05$, again there exist two static bubbles of which $\theta_{5\ell}$ gives the stable one. In this case since $\theta_{5\max}$ is large, it will be easier to realize the closed string tachyon condensation without much requirement on Q_5/L^2 unlike in the previous case. Note that once the condition $Q_5/L^2 < C_{5\max}$ is satisfied, the corresponding condition for the strings should also be satisfied automatically. However, now $\theta_{1\max}$ is vanishingly small, so, let us discuss the values of the string parameters in a slightly more detail. We know that

$$e^{\theta_{5\ell}} > e^{\theta_{5\max}} \approx 2(2k)^{-\frac{1}{3}} \tag{44}$$

is very large while $\theta_{1\ell} > \theta_{1\max} \approx (2k)^{1/3}/2 \ll 1$ can be still very small or of order one or large depending on how large $\theta_{5\ell}$ is. One thing is clear that in order to satisfy eq.(44) $k^2 \sinh^2 2\theta_{5\ell} > e^{-2\theta_{5\ell}}$. So, let us consider three different situations depending on the value of $k^2 \sinh^2 2\theta_{5\ell}$ for which $\theta_{1\ell}$ could be either very small or of order one or very large, namely, (a) $e^{-2\theta_{5\ell}} < k^2 \sinh^2 2\theta_{5\ell} \ll 1$, (b) $k^2 \sinh^2 2\theta_{5\ell} = A \sim 1$, (c) $k^2 \sinh^2 2\theta_{5\ell} \gg 1$.

Let us consider (a) and set

$$k^2 \sinh^2 2\theta_{5\ell} \approx \frac{k^2}{4} e^{4\theta_{5\ell}} = B e^{-2\theta_{5\ell}}, \tag{45}$$

where B is a parameter whose value will be determined from the condition (44). From (45) we obtain

$$e^{\theta_{5\ell}} = 2^{2/3} B^{1/6} (2k)^{-1/3} \gg 1. \tag{46}$$

Now it can be checked that for (44) to be satisfied the parameter B must be restricted as,

$$B > 4. \tag{47}$$

It is not difficult to see that the condition (47) on the B-parameter is just another representation of the condition $Q_5/L^2 < C_{5\max}$ for the existence of static bubble. In fact, if we use the latter condition we can recover the restriction on the B-parameter as follows. Let us first rewrite Q_5/L^2 given in (38) using (45) and then use (46) to get,

$$\frac{Q_5}{L^2} = \frac{1}{2\pi^2}\left[1 - \frac{1+\frac{B}{8}}{(2B)^{\frac{1}{3}}}(2k)^{\frac{2}{3}}\right] < C_{5\max} \approx \frac{1}{2\pi^2}\left[1 - \frac{3}{4}(2k)^{\frac{2}{3}}\right], \tag{48}$$

where in the last step we have used (41). We thus get from (48), $(4+B/2)/3 > (2B)^{1/3}$. This is consistent only if $B = 4(1+\epsilon)$ with $\epsilon > 0$. Now $\theta_{1\ell}$ can be determined from the following,

$$\sinh 2\theta_{1\ell} = k \sinh 2\theta_{5\ell} \approx \frac{k}{2} e^{2\theta_{5\ell}} = \frac{1}{2}(4Bk)^{\frac{1}{3}} \ll 1, \tag{49}$$

where in the last inequality we have used $Bk \ll 2$ which follows from $Be^{-2\theta_{5l}} \ll 1$, (44) and $B > 4$. So, (49) implies small $\theta_{1\ell}$ which has the form,

$$\theta_{1\ell} = \frac{1}{4}(4Bk)^{\frac{1}{3}} \ll 1. \tag{50}$$

We now come to situation (b), where $k^2 \sinh^2 2\theta_{5\ell} = A \sim 1$. In this case we can solve (38) to obtain $A = [L^4/(\pi^4 Q_5^2)](1 - 2\pi^2 Q_5/L^2)$. This can now be used to obtain

$$e^{\theta_{5\ell}} = \frac{2L}{\pi\sqrt{Q_5}} \left(1 - \frac{2\pi^2 Q_5}{L^2}\right)^{\frac{1}{4}} (2k)^{-\frac{1}{2}} \gg 1. \tag{51}$$

We therefore have $\theta_{1\ell}$ as,

$$\sinh 2\theta_{1\ell} = k \sinh 2\theta_{5\ell} = \frac{k}{2} e^{2\theta_{5\ell}} = \frac{L^2}{\pi^2 Q_5} \left(1 - \frac{2\pi^2 Q_5}{L^2}\right)^{\frac{1}{2}} = \text{finite} \tag{52}$$

from which we can now solve to get

$$e^{\theta_{1\ell}} = \frac{L}{\pi\sqrt{Q_5}} \left(1 - \frac{2\pi^2 Q_5}{L^2}\right)^{\frac{1}{4}} \left[1 + \sqrt{1 + \frac{\pi^4 Q_5^2}{L^4}\left(1 - \frac{2\pi^2 Q_5}{L^2}\right)^{-1}}\right]^{\frac{1}{2}}. \tag{53}$$

Next we consider situation (c) for which $k^2 \sinh^2 2\theta_{5\ell} \gg 1$. In this case we get from (38),

$$\frac{Q_5}{L^2} \approx \frac{1}{\pi^2 k \sinh 2\theta_{5\ell}} \approx \frac{2}{\pi^2 k e^{2\theta_{5\ell}}} \quad \Rightarrow \quad e^{\theta_{5\ell}} = \frac{2L}{\pi\sqrt{Q_5}}(2k)^{-\frac{1}{2}} \gg 1. \tag{54}$$

Therefore from $\sinh 2\theta_{1\ell} = k \sinh 2\theta_{5\ell} = L^2/(\pi^2 Q_5) \gg 1$, we have

$$\begin{aligned} e^{\theta_{1\ell}} &= \frac{L}{\pi\sqrt{Q_5}} \left[1 + \sqrt{1 + \frac{\pi^4 Q_5^2}{L^4}}\right]^{\frac{1}{2}} \\ &\approx \sqrt{\frac{2}{Q_5}} \frac{L}{\pi} \left(1 + \frac{\pi^4 Q_5^2}{8L^4}\right) \gg 1 \end{aligned} \tag{55}$$

We thus conclude that for $k \ll 1$, stable static bubble can be obtained but in this case even though $\theta_{5\ell}$ is always large $\theta_{1\ell}$ could be small, large or of order 1 depending on how large the value of $\theta_{5\ell}$ is.

The case of $k \gg 1$ can be discussed in a similar fashion if we set $k' = 1/k$. Then here $\theta_{1\ell}$ takes the role of $\theta_{5\ell}$ in the previous case and vice-versa. So, we will not repeat the discussion here. Thus we have seen how the initial data analysis helps us to discuss the stability of the final bubble in various cases.

4 Physical interpretation of the interpolations

In the previous section we have seen by initial data analysis that under certain circumstances, the black F/NS5 or D1/D5 can indeed make a transition to locally stable static bubbles. In these cases the interpolating solutions described in section 2 make sense as the final bubbles are classically stable and do not evolve further. In this section we will try to give a physical interpretation to these interpolations as the perturbative stringy process of closed string tachyon condensation. As we will see if the closed string tachyon condensation is the possible mechanism for the transition from black solution to KK BON, several conditions have to be satisfied. We will discuss non-susy F/NS5 case first and then the case of non-susy D1/D5.

4.1 Non-susy F/NS5

The classical supergravity solution interpolating between black F/NS5 and KK BON is given in (16). This interpolation can be regarded as a transition from black F/NS5 to KK BON under certain conditions. Since this is a topology changing transition like what happens for stringy process of closed string tachyon condensation, the latter can be taken as a possible mechanism for the transition as we will argue. We will consider the closed string tachyon condensation to occur on the horizon, where the space-time curvature must be small compared to the string length otherwise the supergravity description will break down. The black F/NS5 supergravity configuration is given in (21) where the coordinate x^5 is compact. In order to have a closed string tachyon condensation the size of the x^5 circle must satisfy

$$L = l_s \cosh\theta_1 \tag{56}$$

where l_s is the fundamental string length. The size of the horizon and the two charges associated with the solution can be written as,

$$Z = \rho_0 \cosh\theta_5, \qquad Q_1 = \rho_0^2 \sinh 2\theta_1, \qquad Q_5 = \rho_0^2 \sinh 2\theta_5. \tag{57}$$

The corresponding KK BON solution is given in (23). We have found that to avoid conical singularity at ρ_0, x^5 must be periodic with a periodicity given in (24). We denote the various bubble quantities with a subscript 'b' and also, since the parameters for the two solutions need not be the same, we denote the bubble parameters with a 'tilde'. So, the periodicity, size of the bubble, and the fluxes can be written as follows:

$$L_b = 2\pi\tilde\rho_0 \cosh\tilde\theta_1 \cosh\tilde\theta_5, \quad Z_b = \tilde\rho_0 \cosh\tilde\theta_5, \quad Q_{1b} = \tilde\rho_0^2 \sinh 2\tilde\theta_1, \quad Q_{5b} = \tilde\rho_0^2 \sinh 2\tilde\theta_5. \tag{58}$$

Now if black F/NS5 solution makes a transition to KK BON, then the quantities given above for the two solutions must be equated. In fact we must have $Q_1 = Q_{1b}$, $Q_5 = Q_{5b}$ as exact relations due to the charge conservation. Also we must have $L = L_b$ as exact relation as well since this is the asymptotic radius of x^5-circle and tachyon condensation occurs on the horizon as a local process. However, the radius of the horizon and the bubble size will have only their leading order (i.e., the classical part) equal since the tachyon condensation occurs on the horizon and there may be some quantum corrections. This is evident that θ_5 and $\tilde{\theta}_5$ always remain large for the transition as will be demonstrated in the following and the ratio of the sub-leading order over the leading order is of the order of $\mathcal{O}(e^{-2\theta_5} \text{ or } e^{-2\tilde{\theta}_5})$, i.e., exponentially small. For this reason, we can have only $Z \approx Z_b \gg l_s$.

It is clear from (56), that θ_1 is very large and so, we have,

$$e^{\theta_1} \approx \frac{2L}{l_s}. \tag{59}$$

Also from the definition of k as well as (57) we have

$$\sinh 2\theta_5 \approx \frac{2L^2}{l_s^2 k}, \qquad \rho_0 \approx \sqrt{\frac{kQ_5}{2}}\frac{l_s}{L}. \tag{60}$$

Now let us first assume $k \sim 1$, implying that θ_5 is also very large. So, the θ_5 equation in (60) simplifies to $e^{\theta_5} \approx 2L/(\sqrt{k}l_s)$. Further, from $Q_5 = Q_{5b}$ and $Z \approx Z_b$ we get $\tanh\theta_5 \approx \tanh\tilde{\theta}_5$, which implies $\tilde{\theta}_5$ is very large and so, $\tilde{\theta}_1$ is also very large. Moreover, using these as well as $L = L_b \gg l_s$, $Q_1 = Q_{1b}$ in addition, we have,

$$e^{\theta_1} \gg e^{\tilde{\theta}_1}, \quad \rho_0 \ll \tilde{\rho}_0, \quad e^{\theta_5} \gg e^{\tilde{\theta}_5}, \quad \text{and} \quad \rho_0 e^{\theta_5} \approx \tilde{\rho}_0 e^{\tilde{\theta}_5} \gg l_s. \tag{61}$$

We already discussed that for $k \sim 1$, the stable static bubble corresponds to (see eq.(43))

$$e^{\tilde{\theta}_{1\ell}} \sim \sqrt{\frac{2}{Q_5}}\frac{L}{\pi}, \qquad e^{\tilde{\theta}_{5\ell}} \sim \sqrt{\frac{2}{kQ_5}}\frac{L}{\pi} \tag{62}$$

with $Q_5/L^2 \ll C_{5\max} \approx 0.02$ and from (58), we have

$$\tilde{\rho}_0 = \pi\sqrt{kQ_5}\sqrt{\frac{Q_5}{L^2}}. \tag{63}$$

Given $\sqrt{Q_5} \gg l_s$, along with black F/NS5 parameters (59) and (60) and also the bubble parameters (62), (63), it is obvious that all the conditions in (61) can be satisfied. So, the transition can indeed be caused by the closed string tachyon condensation.

Next we consider $k \ll 1$ case. Here also both θ_1 and θ_5 are quite large and so as in the previous case (59) and (60) with $\sinh 2\theta_5$ replaced by $e^{2\theta_5}/2$ hold. As before in this

case also $\tilde{\theta}_{5\ell}$ is always large (which follows from $Q_5 = Q_{b5}$ and $Z \approx Z_b$), but $\tilde{\theta}_{1\ell}$ can be small, of order one or large, as we discussed earlier, depending on how large the value of $\tilde{\theta}_{5\ell}$ is. For small $\tilde{\theta}_{1\ell}$, we have from (46) and (50)

$$e^{\tilde{\theta}_{5\ell}} = 2^{\frac{2}{3}} B^{\frac{1}{6}} (2k)^{-\frac{1}{3}}, \qquad \tilde{\theta}_{1\ell} = \frac{1}{4}(4Bk)^{\frac{1}{3}} \tag{64}$$

and from (58) we have,

$$\tilde{\rho}_0 = (2B)^{-\frac{1}{6}} \sqrt{Q_5} (2k)^{\frac{1}{3}}. \tag{65}$$

The condition for the closed string tachyon condensation in this case is slightly different (since $\tilde{\theta}_{1\ell}$ is small) from the previous case (61) as,

$$e^{\theta_1} \gg 4\pi \cosh \tilde{\theta}_1, \quad \rho_0 \ll \tilde{\rho}_0, \quad e^{\theta_5} \gg e^{\tilde{\theta}_5}, \quad \text{and} \quad \rho_0 e^{\theta_5} \approx \tilde{\rho}_0 e^{\tilde{\theta}_5} \gg l_s. \tag{66}$$

It can be verified that all these conditions can be satisfied using (59) and (60) for black F/NS5 and (64) and (65) for the bubble provided

$$\sqrt{Q_5} \gg l_s, \tag{67}$$

which can be satisfied easily.

For the case of finite $\tilde{\theta}_{1\ell}$, as discussed earlier (see (51) and (53)), we now have

$$\begin{aligned} e^{\tilde{\theta}_{5\ell}} &= \frac{2L}{\pi\sqrt{Q_5}} \left(1 - \frac{2\pi^2 Q_5}{L^2}\right)^{\frac{1}{4}} (2k)^{-\frac{1}{2}}, \\ \sinh 2\tilde{\theta}_{1\ell} &= \frac{L^2}{\pi^2 Q_5} \left(1 - \frac{2\pi^2 Q_5}{L^2}\right)^{\frac{1}{2}}, \end{aligned} \tag{68}$$

also from (58) we have

$$\tilde{\rho}_0 = \frac{\pi}{\sqrt{2}} \frac{Q_5}{L} \left(1 - \frac{2\pi^2 Q_5}{L^2}\right)^{-\frac{1}{4}} (2k)^{\frac{1}{2}}. \tag{69}$$

The conditions for the closed string tachyon condensation in this case remain the same as in (66). It can be checked that they can be satisfied if we use (68) and (69) provided $\sqrt{Q_5} \gg l_s$.

For the case of large $\tilde{\theta}_{1\ell}$, we have from (54) and (55)

$$\begin{aligned} e^{\tilde{\theta}_{5\ell}} &= \frac{2L}{\pi\sqrt{Q_5}} (2k)^{-\frac{1}{2}}, \\ e^{\tilde{\theta}_{1\ell}} &= \sqrt{\frac{2}{Q_5}} \frac{L}{\pi} \left(1 + \frac{\pi^4 Q_5^2}{8L^4}\right) \end{aligned} \tag{70}$$

with $L^2/(\pi^2 Q_5) \gg 1$. Then from (58) we get,

$$\tilde{\rho}_0 = \sqrt{Q_5}\frac{\pi\sqrt{Q_5}}{L}(2k)^{\frac{1}{2}}. \tag{71}$$

Here the conditions for the closed string tachyon condensation (61) can be satisfied provided $\sqrt{Q_5} \gg l_s$.

We would like to point out that since the occurrence of closed string tachyon condensation requires $\rho_0 \ll \tilde{\rho}_0$, for the supergravity description to remain valid, the string coupling must be small at $\rho = \rho_0$. Putting in the asymptotic string coupling, the dilaton in this case has the form $e^{2\tilde{\phi}} = g_s^2 \bar{G}_5/\bar{G}_1$ and so, its value at $\rho = \rho_0$ is $g_s^2 \cosh^2\theta_5/\cosh^2\theta_1 \approx g_s^2 e^{2\theta_5 - 2\theta_1} \approx g_s^2 Q_5/Q_1$. Note that both θ_1 and θ_5 are large here and so, the hyperbolic functions can be approximated by the exponentials. For F/NS5 solution we can write $Q_1 = (N_1/V_4)\alpha'^3 g_s^2$, and $Q_5 = N_5\alpha'$, where V_4 is the volume covered by NS5-branes transverse to F-strings and $\alpha' = l_s^2$. If we introduce a dimensionless volume by $v_4 = V_4/\alpha'^2$, then $e^{2\tilde{\phi}} = v_4 N_5/N_1$ and so, for this to remain small, an additional condition has to be satisfied which is $v_4 N_5 \ll N_1$ and if we have $v_4 N_5 \gg N_1$, we must go to the S-dual D1/D5 configuration which we will discuss in the next subsection.

Now we come to the case of $k \gg 1$. In this case the string charge dominates. The black F/NS5 parameters remain the same as given in (59) and (60). It is now more proper to use string charge $Q_1 = kQ_5$, instead of Q_5. Note that both Q_1 and Q_5 are supposed to be fixed macroscopic quantities and so $k \gg 1$ implies that k is also supposed to be large but fixed. As discussed in the last paragraph, $Q_1 = kQ_5$ gives $g_s^2 N_1/v_4 = kN_5$. Since $g_s \ll 1$ and $v_4 \geq 1$, large k requires very large N_1/v_4 which is surely more difficult to realize. On the other hand, $k = \sinh 2\theta_1/\sinh 2\theta_5 \approx e^{2\theta_1}/(2\sinh 2\theta_5)$ (since θ_1 is large, being independently determined by (59)). So the largeness of k depends on the value of θ_5 which can now be small, of order unity and large. Therefore a rather large θ_5 can give rise to a fixed large k which should be easier to realize. While the other two scenarios can also be considered, we here limit ourselves to the large θ_5 for the reason just given and for simplicity as well. For now, the closed string tachyon condensation condition is again given by (66). Note that $\tilde{\theta}_5$ continues to be large (due to $\tanh\theta_5 \approx \tanh\tilde{\theta}_5$). On the bubble side, we can repeat the earlier analysis by using string rather than fivebrane, but by replacing $k \to k' = 1/k \ll 1$. Now $\tilde{\theta}_{1\ell}$ is large, but $\tilde{\theta}_{5\ell}$ can be small, of order one or large. As just mentioned only large $\tilde{\theta}_{5\ell}$ is relevant here. What we have done for $k \ll 1$ cases can be borrowed here with the replacements $1 \leftrightarrow 5$ and $k \to 1/k$. With this, we have now on the bubble side,

$$e^{\tilde{\theta}_{1\ell}} = \frac{2L}{\pi\sqrt{Q_1}}\left(\frac{k}{2}\right)^{\frac{1}{2}},$$

$$e^{\tilde{\theta}_{5\ell}} = \sqrt{\frac{2}{Q_1}}\frac{L}{\pi}\left(1+\frac{\pi^4 Q_1^2}{8L^4}\right) \tag{72}$$

with now $L^2/(\pi^2 Q_1) \gg 1$. Then from (58) we get,

$$\tilde{\rho}_0 = \sqrt{Q_1}\frac{\pi\sqrt{Q_1}}{L}\left(\frac{2}{k}\right)^{\frac{1}{2}}. \tag{73}$$

The condition (66) can again be satisfied provided $\sqrt{Q_5} \gg l_s$. This again requires that there be enough number of NS5-branes present even though $Q_1/Q_5 = k \gg 1$. In other words, so long as there are enough number of NS5-branes present, this condition can be satisfied.

In this case since θ_1 is much greater than θ_5, the dilaton will always remain small here and there is no additional condition to be satisfied.

In summary, we have seen that with the presence of F-strings in NS5-brane systems or the presence of NS5-branes in F-string systems makes the transition from the black solution to KK BON possible via the stringy closed string tachyon condensation process.

4.2 Non-susy D1/D5

The classical supergravity solution interpolating between black D1/D5 and KK BON is given in (25). As we have seen in section 3 by initial data analysis, this interpolation can be regarded as a transition from black D1/D5 to KK BON under certain conditions. Note that although we have not explicitly performed the initial data analysis for non-susy D1/D5 solution, but since this is S-dual to F/NS5 solution, the analysis remains the same. Again we will try to give a physical interpretation of this transition as the closed string tachyon condensation. As before, we will consider the closed string tachyon condensation to occur on the horizon, where the space-time curvature must be small compared to the string length otherwise the supergravity description will break down. The black D1/D5 supergravity configuration is given in (27) where the coordinate x^5 is compact. In order to have a closed string tachyon condensation the size of the x^5 circle must satisfy

$$L = \bar{l}_s \cosh^{\frac{1}{2}}\theta_1 \cosh^{\frac{1}{2}}\theta_5. \tag{74}$$

Here we have put a 'bar' on l_s to distinguish that from the previous section and they are related by S-duality. The size of the horizon and the two charges associated with the solution can be written as,

$$Z = \rho_0 \cosh^{\frac{1}{2}}\theta_1 \cosh^{\frac{1}{2}}\theta_5, \qquad Q_1 = \rho_0^2 \sinh 2\theta_1, \qquad Q_5 = \rho_0^2 \sinh 2\theta_5. \tag{75}$$

The corresponding KK BON solution is given in (29). We have found that to avoid conical singularity at ρ_0, x^5 must be periodic with a periodicity given in (24). As before we denote the various bubble quantities with a subscript 'b' and also, the bubble parameters with a 'tilde'. So, the periodicity, the size of the bubble, and the fluxes can be written as follows:

$$\begin{aligned} L_b &= 2\pi\tilde{\rho}_0 \cosh\tilde{\theta}_1 \cosh\tilde{\theta}_5, \quad Z_b = \tilde{\rho}_0 \cosh^{\frac{1}{2}}\tilde{\theta}_1 \cosh^{\frac{1}{2}}\tilde{\theta}_5, \quad Q_{1b} = \tilde{\rho}_0^2 \sinh 2\tilde{\theta}_1, \\ Q_{5b} &= \tilde{\rho}_0^2 \sinh 2\tilde{\theta}_5. \end{aligned} \tag{76}$$

Since $L \gg \bar{l}_s$, we get from (74)

$$\cosh\theta_1 \cosh\theta_5 \gg 1. \tag{77}$$

We have discussed all the possible cases in a bit detail in the previous subsection and we could do that in this subsection as well, but instead we will consider only the case where all the angles are large. This is not completely unreasonable since given Q_1 and Q_5, we can always insist large L to make all the angles $\theta_{1,5}$ and $\tilde{\theta}_{1,5}$ large. So, for simplicity in this subsection we will consider only the case where all angles are large.

Now since the angles are large we have from (74) and (75),

$$e^{\theta_1+\theta_5} = \left(\frac{2L}{\bar{l}_s}\right)^2, \quad \frac{L}{Z} = \frac{\bar{l}_s}{\rho_0}, \quad Q_1 = \frac{\rho_0^2}{2} e^{2\theta_1}, \quad Q_5 = \frac{\rho_0^2}{2} e^{2\theta_5} \tag{78}$$

and from (76),

$$L_b = \frac{\pi\tilde{\rho}_0}{2} e^{\tilde{\theta}_1+\tilde{\theta}_5}, \quad \frac{L_b}{Z_b} = \pi e^{\frac{\tilde{\theta}_1+\tilde{\theta}_5}{2}}, \quad Q_{1b} = \frac{\tilde{\rho}_0^2}{2} e^{2\tilde{\theta}_1}, \quad Q_{5b} = \frac{\tilde{\rho}_0^2}{2} e^{2\tilde{\theta}_5}. \tag{79}$$

Now for closed string tachyon condensation occurring on the horizon, as mentioned earlier, we must equate,

$$L = L_b, \qquad Z \approx Z_b, \qquad Q_1 = Q_{1b}, \qquad \text{and} \qquad Q_5 = Q_{5b}. \tag{80}$$

Using the relation $L/Z \approx L_b/Z_b$, we have

$$\rho_0 = \frac{\bar{l}_s}{\pi} e^{-\frac{\tilde{\theta}_1+\tilde{\theta}_5}{2}} \ll \bar{l}_s \tag{81}$$

Further from the expression of Z given in (75) and the requirement of $Z \gg \bar{l}_s$ for small curvature at the horizon, we have, using (81),

$$Z = \frac{\bar{l}_s}{2\pi} \left(\frac{e^{\theta_1+\theta_5}}{e^{\tilde{\theta}_1+\tilde{\theta}_5}}\right)^{\frac{1}{2}} \gg \bar{l}_s. \tag{82}$$

Eq.(82) implies $e^{\theta_1+\theta_5} \gg e^{\tilde{\theta}_1+\tilde{\theta}_5}$. Also from $Q_1/Q_5 = Q_{1b}/Q_{5b}$, we obtain $e^{\theta_1-\theta_5} = e^{\tilde{\theta}_1-\tilde{\theta}_5}$ and combining these two relations we have

$$e^{\theta_1} \gg e^{\tilde{\theta}_1}, \qquad e^{\theta_5} \gg e^{\tilde{\theta}_5}. \tag{83}$$

Using (83) and the charge conservation we get

$$\rho_0 \ll \tilde{\rho}_0. \tag{84}$$

We can now solve θ_1, θ_5 and ρ_0 from (78) as,

$$e^{\theta_1} = \frac{2L}{\bar{l}_s}\left(\frac{Q_1}{Q_5}\right)^{\frac{1}{4}}, \quad e^{\theta_5} = \frac{2L}{\bar{l}_s}\left(\frac{Q_5}{Q_1}\right)^{\frac{1}{4}}, \quad \rho_0 = \frac{\bar{l}_s}{\sqrt{2}}\left(\frac{Q_1 Q_5}{L^4}\right)^{\frac{1}{4}} \tag{85}$$

and solve $\tilde{\theta}_1$, $\tilde{\theta}_5$ and $\tilde{\rho}_0$ from (79) as,

$$e^{\tilde{\theta}_1} = \frac{\sqrt{2}}{\pi}\sqrt{\frac{L^2}{Q_5}}, \quad e^{\tilde{\theta}_5} = \frac{\sqrt{2}}{\pi}\sqrt{\frac{L^2}{Q_1}}, \quad \tilde{\rho}_0 = \pi L\left(\frac{Q_1 Q_5}{L^4}\right)^{\frac{1}{2}}. \tag{86}$$

It can now be easily checked that with the above solutions the conditions for the closed string tachyon condensation (80), (83) and (84) can be satisfied provided,

$$(Q_1 Q_5)^{\frac{1}{4}} \gg \frac{\bar{l}_s}{\sqrt{2}\,\pi}. \tag{87}$$

Let us try to understand the relation (87) in more detail here. Writing $Q_1 = \bar{g}_s(N_1/\bar{v}_4)\bar{\alpha}'$ and $Q_5 = \bar{g}_s N_5 \bar{\alpha}'$, where $\bar{g}_s$ is the asymptotic string coupling and $\bar{\alpha}' = \bar{l}_s^2$, with $\bar{l}_s$ the fudamental string length. Also $\bar{v}_4 = V_4/\bar{\alpha}'^2$. As before we have put a 'bar' in string coupling to distinguish from that used in the previous subsection and they are related by S-duality. Note that the total volume V_4 remains unchanged. Now using these Q_1 and Q_5, the relation (87) becomes,

$$\frac{N_1 \bar{g}_s^2}{\bar{v}_4} \gg \frac{1}{4\pi^4 N_5}. \tag{88}$$

Let us now look at the dilaton which must be small at $\rho = \rho_0$. The dilaton has the form (see eq.(29)) $e^{2\phi} = \bar{g}_s^2 \bar{G}_1/\bar{G}_5 = \bar{g}_s^2 \cosh^2\theta_1/\cosh^2\theta_5 \approx \bar{g}_s^2 Q_1/Q_5$ for large θ's. Using the form of Q_1 and Q_5 given above we get

$$e^{2\phi} = \frac{\bar{g}_s^2 N_1}{\bar{v}_4 N_5} \ll 1 \quad \Rightarrow \quad \bar{g}_s^2 N_1 \ll \bar{v}_4 N_5. \tag{89}$$

Now using S-duality $\bar{g}_s = 1/g_s$, $\bar{\alpha}' = g_s\alpha'$ and $V_4 = v_4\alpha'^2 = \bar{v}_4\bar{\alpha}'^2 = \bar{v}_4 g_s^2 \alpha'^2 \Rightarrow \bar{v}_4 g_s^2 = v_4$, we get from (89)

$$N_5 \gg \frac{N_1}{v_4}. \tag{90}$$

This is the precise condition for using the S-dual D1/D5 description as mentioned in the previous subsection. This relation is also consistent with the condition of closed string tachyon condensation we obtained in (88). In fact combining (88) and (90) we have

$$N_5 \gg \frac{N_1}{v_4} \gg \frac{1}{4\pi^4 N_5}. \tag{91}$$

So, everything fits nicely.

5 Special cases

In this section we will mention two special cases of our general non-susy F/NS5 solution and non-susy D1/D5 solution for the case of $k = 1$ as discussed in the previous two sections. First we will show how the two-charge F-string discussed by Horowitz can be obtained as a special case of the non-susy F/NS5 solution and then we show how the interpolating solution between AdS$_3$ black hole and the global AdS$_3$ can be obtained as a special case of the non-susy D1/D5 solution.

5.1 Two-charge F-string

Two charge F-string solution considered by Horowitz can be seen to arise as a special case from the general non-susy F/NS5 solution we obtained in (16). The two-charge F-string solution is a six-dimensional string solution which can be obtained if we simply restrict the parameters δ_1 and δ_2 as $\delta_1 = 4\delta_2$ in (16) and compactify the directions $x^1, \ldots, x^4$ on T^4. The solution then reduces to

$$\begin{aligned}
ds^2_{6,\mathrm{str}} &= G_1^{-1} f^{\frac{\hat{\alpha}_1}{2}} \left(-dt^2 + f^{3\delta_2}(dx^5)^2\right) + G_5 f^{-\frac{\hat{\alpha}_5}{2}+\frac{3\delta_2}{2}+\frac{1}{2}} \left(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2\right),\\
e^{2\tilde{\phi}} &= G_5 G_1^{-1} f^{-\frac{\hat{\alpha}_5}{2}+\frac{\hat{\alpha}_1}{2}+3\delta_2},\\
B_{[2]} &= \frac{1}{2}\sinh 2\theta_1 \left(\frac{1 - f^{\frac{\hat{\alpha}_1+\hat{\beta}_1}{2}}}{G_1}\right) dt \wedge dx^5, \qquad H_{[3]} = b\mathrm{Vol}(\Omega_3),
\end{aligned} \tag{92}$$

where the parameter relations following from (11) are given as $\hat{\alpha}_1 - \hat{\beta}_1 = 6\delta_2$, $\hat{\alpha}_5 - \hat{\beta}_5 = -6\delta_2$ and $(\hat{\alpha}_1 + \hat{\beta}_1)^2 + (\hat{\alpha}_5 + \hat{\beta}_5)^2 = 12(1 - 3\delta_2^2)$. The solution (92) is the $D = 6$ non-susy two charge F-string solution and is characterized by five independent parameters $(\hat{\alpha}_1 + \hat{\beta}_1)$, $(\hat{\alpha}_5 + \hat{\beta}_5)$, ρ_0, θ_1 and δ_2. In the above we have put $\theta_1 = \theta_5$ as Horowitz for simplicity. It can be easily checked that the solution (92) interpolates between two-charge

black F-string and KK BON if we vary the parameters keeping ρ_0 and θ_1 fixed. Indeed if we choose

$$\hat{\alpha}_1 + \hat{\beta}_1 = 2, \qquad \hat{\alpha}_5 + \hat{\beta}_5 = 2, \qquad \delta_2 = -\frac{1}{3} \tag{93}$$

(which implies from the parameter relations given above that $\hat{\alpha}_1 = 2$, $\hat{\beta}_1 = 0$ and $\hat{\alpha}_5 = 0$, $\hat{\beta}_5 = 2$), then the configuration (92) takes the form,

$$\begin{aligned}
ds^2_{6,\mathrm{str}} &= \bar{G}_1^{-1}\left(-f dt^2 + (dx^5)^2\right) + \bar{G}_1\left(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2\right),\\
e^{2\tilde{\phi}} &= 1,\\
B_{[2]} &= \coth\theta_1\left(\frac{\bar{G}_1 - 1}{\bar{G}_1}\right) dt\wedge dx^5, \qquad H_{[3]} = b\mathrm{Vol}(\Omega_3),
\end{aligned} \tag{94}$$

where $b = \rho_0^2\sinh\theta_1$ and G_1 and G_5 are now equal and take the form $G_{1,5} \to \bar{G}_1 (= \bar{G}_5) = 1 + \rho_0^2\sinh^2\theta_1/\rho^2$. (94) is precisely the two charge black F-string described in [1]. On the other hand, if we choose,

$$\hat{\alpha}_1 + \hat{\beta}_1 = 2, \qquad \hat{\alpha}_5 + \hat{\beta}_5 = 2, \qquad \delta_2 = \frac{1}{3} \tag{95}$$

(which implies from the parameter relations given above that $\hat{\alpha}_1 = 0$, $\hat{\beta}_1 = 2$ and $\hat{\alpha}_5 = 2$, $\hat{\beta}_5 = 0$), then the configuration (92) takes the form,

$$\begin{aligned}
ds^2_{6,\mathrm{str}} &= \bar{G}_1^{-1}\left(-dt^2 + f(dx^5)^2\right) + \bar{G}_1\left(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2\right),\\
e^{2\tilde{\phi}} &= 1,\\
B_{[2]} &= \coth\theta_1\left(\frac{\bar{G}_1 - 1}{\bar{G}_1}\right) dt\wedge dx^5, \qquad H_{[3]} = b\mathrm{Vol}(\Omega_3).
\end{aligned} \tag{96}$$

This is precisely the KK BON solution given in [1]. Here in order to avoid the conical singularity at $\rho = \rho_0$, the coordinate x^5 must be periodic with period $L = 2\pi\rho_0\cosh^2\theta_1$. It is therefore clear that (92) is the solution which interpolates between the two charge black F-string to KK BON by continuously varying the parameters $\hat{\alpha}_{1,5}$, $\hat{\beta}_{1,5}$ and δ_2 and there is no need to take the double Wick rotation. As we have seen for the general case of non-susy F/NS5 solution in sections 3 and 4, that the bubble here could be stable and static and the interpolation can be understood as a physical process of closed string tachyon condensation.

5.2 AdS$_3$ black hole

The interpolation from the AdS$_3$ black hole to global AdS$_3$ can be seen to arise as a special case of non-susy D1/D5 system we obtained in (25). The general intersecting

non-susy D1/D5 system with chargeless D0-branes in Schwarzschild-like coordinate is given in (25). If we fix $\hat{\alpha}_{1,5} + \hat{\beta}_{1,5} = 2$ keeping δ_1 and δ_2 arbitrary (so, individually $\hat{\alpha}_1$, $\hat{\alpha}_2$, or $\hat{\beta}_1$, $\hat{\beta}_2$ remain arbitrary) then the functions $G_{1,5}$ will always have the forms $\bar{G}_{1,5} = \cosh^2\theta_{1,5} - f\sinh^2\theta_{1,5} = 1 + \rho_0^2\sinh^2\theta_{1,5}/\rho^2$. Note that this restriction of $\hat{\alpha}_{1,5}+\hat{\beta}_{1,5}$ is necessary to have the AdS structure unlike in the previous case. Let us also put $\theta_1 = \theta_5$ for simplicity as in the previous case. Now if we restrict the radial variable in the region $\rho_0 \leq \rho \ll \rho_0\sinh\theta_1$, then we have $\bar{G}_1 = \bar{G}_5 \approx \rho_0^2\sinh^2\theta_1/\rho^2 \equiv R^2/\rho^2$. With these restrictions the solution (25) reduces to,

$$\begin{aligned}
ds^2_{\rm str} &= \frac{\rho^2}{R^2}\left(-f^{\frac{1}{2}-\frac{3\delta_1}{8}}dt^2 + f^{\frac{1}{2}+\frac{\delta_1}{8}+\delta_2}(dx^5)^2\right) + \frac{R^2}{\rho^2}f^{-\frac{\delta_1}{8}+\frac{\delta_2}{2}}\left(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2\right) + \sum_{i=1}^{4}(dx^i)^2,\\
e^{2\phi} &= f^{-\frac{\delta_1}{4}+\delta_2},\\
F_{[3]} &= -2\frac{\rho}{R^2}\coth\theta_1\, d\rho\wedge dt\wedge dx^5,\\
F_{[7]} &= -2\frac{\rho}{R^2}\coth\theta_1\, d\rho\wedge dt\wedge dx^1\wedge dx^2\wedge dx^3\wedge dx^4\wedge dx^5.
\end{aligned}\tag{97}$$

The parameter relations (11) in this case take the forms,

$$\begin{aligned}
&\hat{\alpha}_1 = 1 - \frac{3}{4}\delta_1, \quad \hat{\alpha}_5 = 1 + \frac{3}{4}\delta_1, \quad \hat{\beta}_1 = 1 + \frac{3}{4}\delta_1, \quad \hat{\beta}_5 = 1 - \frac{3}{4}\delta_1,\\
&\frac{3}{8}\delta_1^2 + 3\delta_2^2 - 1 = 0.
\end{aligned}\tag{98}$$

Now it can be checked that if we choose

$$\delta_1 = -\frac{4}{3} \qquad \text{and} \quad \delta_2 = -\frac{1}{3}\tag{99}$$

implying from (98) $\hat{\alpha}_1 = 2$, $\hat{\beta}_1 = 0$, $\hat{\alpha}_5 = 0$, $\hat{\beta}_5 = 2$, then the above solution (97) reduces to

$$\begin{aligned}
ds^2_{\rm str} &= \frac{\rho^2}{R^2}\left(-f dt^2 + (dx^5)^2\right) + \frac{R^2}{\rho^2}\left(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2\right) + \sum_{i=1}^{4}(dx^i)^2,\\
e^{2\phi} &= 1,\\
F_{[3]} &= -2\frac{\rho}{R^2}\coth\theta_1\, d\rho\wedge dt\wedge dx^5,\\
F_{[7]} &= -2\frac{\rho}{R^2}\coth\theta_1\, d\rho\wedge dt\wedge dx^1\wedge dx^2\wedge dx^3\wedge dx^4\wedge dx^5.
\end{aligned}\tag{100}$$

This is the $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$ charged black hole solution. On the other hand, if we choose

$$\delta_1 = \frac{4}{3}, \qquad \text{and} \quad \delta_2 = \frac{1}{3}\tag{101}$$

implying from (98) $\hat{\alpha}_1 = 0$, $\hat{\beta}_1 = 2$, $\hat{\alpha}_5 = 2$, $\hat{\beta}_5 = 0$, then the above solution (97) reduces to

$$\begin{aligned}
ds^2_{\rm str} &= \frac{\rho^2}{R^2}\left(-dt^2 + f(dx^5)^2\right) + \frac{R^2}{\rho^2}\left(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2\right) + \sum_{i=1}^{4}(dx^i)^2, \\
e^{2\phi} &= 1, \\
F_{[3]} &= -2\frac{\rho}{R^2}\coth\theta_1\, d\rho\wedge dt\wedge dx^5, \\
F_{[7]} &= -2\frac{\rho}{R^2}\coth\theta_1\, d\rho\wedge dt\wedge dx^1\wedge dx^2\wedge dx^3\wedge dx^4\wedge dx^5.
\end{aligned} \tag{102}$$

In this case, the conical singularity at $\rho = \rho_0$ can be avoided if the coordinate x^5 has the periodicity

$$L = 2\pi\rho_0 \sinh^2\theta_1 \tag{103}$$

This is the global $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$ solution. We thus find that the classical solution (97) interpolates nicely between the AdS_3 black hole and the global AdS_3 by varying some parameters characterizing the solution. Again as we have seen for the general non-susy D1/D5 solution in sections 3 and 4, the bubble here could be stable and static and the transition can be understood to be caused by the closed string tachyon condensation.

6 Discussion

It has been argued by Horowitz [1] that black strings under certain conditions can decay into KK BON by a perturbative stringy process called closed string tachyon condensation. The two ends of this process (black string and KK BON) are well described by the classical supergravity configuration as is well-known. In this paper we have constructed the more general supergravity solution describing non-susy D1/D5 and non-susy F/NS5 solutions. These solutions were shown to interpolate smoothly between black or non-extremal solution and the KK BON by varying some parameters characterizing the solutions from one set of values to another. Horowitz's two charge black F-string and AdS_3 black hole were shown to arise as special cases of these general solutions. We have performed a time symmetric general bubble initial data analysis to argue that the final bubble configurations could, under certain conditions, be locally stable and static such that they do not evolve further perturbatively. We have further shown that this transition can be physically interpreted, under certain circumstances, as the perturbative stringy process of closed string tachyon condensation and the interpolating solutions in turn could be thought of as models of such process.

However, there could be problems with these interpretations to which we turn next.

- The interpolating solutions (16), (25), are well-behaved only at the two end points, the black solutions have singularities masked by regular horizons and the bubble solutions are completely regular. However, the solutions have naked singularities at $\rho = \rho_0$ for all the intermediate points. If they represent the closed string tachyon condensation how do we interpret the intermediate stages which are singular?

- It is known [6] that the closed string tachyon condensation on the horizon is a quick process which occurs at the time scale of the order of string scale. But the interpolation by a classical supergravity description is a continuous process which is a slow adiabatic process. How can a violent process of closed string tachyon condensation be described by supergravity?

These points have been discussed for black Dp-branes in [7]. We will briefly mention them here. For the first point, we remark that the intermediate solutions with parameters other than those given in (18), (20), (22), (26), and (28), corresponding to the two end points of various solutions, are all regular in the region $\rho_0 < \rho \leq \infty$ and the naked singularity at $\rho = \rho_0$ reflects our inability to describe the system classically where the violent quantum process of closed string tachyon condensation is occurring. It is very likely that quantum mechanically there are no singularities, but classically we do not have a good description in general for $\rho \leq \rho_0$. To an observer far away from the core region, only the long range force would appear and so, we have a classical description of the dynamics there. The long distance description is just the family of intermediate solutions with naked singularities. The singularities are actually the artifact as their appearance is due to the extrapolation of the solutions valid only at long distance to the region where the description is invalid and where perhaps we have quantum process without any singularity.

For the second point we remark that it is true that the time scale for the completion of the closed string tachyon condensation process at the horizon is of the order of string scale, but this time is the local proper time. Due to the red-shift factor in front of the time coordinate it is clear that for an observer at infinity (with respect to whom the ADM mass is measured) the time taken for the completion of this process would be infinite and so for this observer the closed string tachyon condensation would appear as a slow, adiabatic process which can be well described by a supergravity with smooth interpolation. This is what our interpolating solutions describe and have no conflict with the observations made in [6].

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